A Construction of a Mathematical Literacy Diagnostic Test for 9th Grade Students in Chiang Mai Province

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Abstract

This study sought to investigate the current state of deficiencies in mathematical literacy and to construct a specific diagnostic test for 9th grade students in Chiang Mai Province. This study was conducted in two phases. In Phase 1, the researchers conducted a PISA-based survey to identify literacy gaps. The instrument focuses on three core mathematical problem-solving processes: formulating mathematical situations, employing mathematical concepts and procedures for solutions, and interpreting and evaluating mathematical outcomes. The preliminary test was administered to a sample of 385 students via stratified random sampling. The deficiencies of 22 specific areas were revealed from 10 identified behavioral indicators. Phase 2, a four-tier mathematical literacy diagnostic test, consisted of four distinct sections: the first tier is the student's answer; the second tier is confidence in that answer; the third tier is the reasons for the answers; and the fourth tier is confidence in the reasoning provided. This diagnostic test was implemented with a sample group of 100 students. Three versions were developed: Version 1 (Formulating, 15 items), Version 2 (Employing, 20 items), and Version 3 (Interpreting, 15 items). All the test items showed difficulty indices ranging from 0.21 to 0.79 and discrimination indices between 0.20 and 0.72, respectively. In addition, all versions revealed high reliability coefficients of .86, .89, and .78, respectively, and concurrent validity coefficients were established at .82, .82, and .83, respectively.

Keywords: Diagnostic Test, Mathematical Literacy, Mathematical Problem-Solving Process

Background and Statement of the problem

Mathematical literacy serves as a foundational pillar for developing human cognitive systems, offering the structured framework necessary for logical reasoning and creative problem-solving. In addition to the theoretical basis, this competency enables individuals to generate accurate predictions and make informed decisions when navigating the complexities of daily life. Consequently, mathematical literacy is not confined to classroom content; it is defined by the practical application of knowledge to real-world scenarios. This requires a sophisticated integration of skills, including the ability to formulate problems and evaluate situations—an interplay of knowledge and higher-order competencies. To this end, mathematics instruction must be structured to cultivate these specific growth areas through learning processes that prioritize holistic student development. In the context of Thailand, basic education is directed by the Basic Education Core Curriculum B.E. 2551 (Revised Edition B.E. 2560), which manages mathematics into three learning standards. Currently, the learning outcomes of these curriculum indicators were under the benchmarks. It is evident that the national test results for 9th grade students from the academic years of 2022, 2023, and 2024 had mean scores of 24.39%, 25.38%, and 26.53%, respectively. These results show that national averages are still stuck below 30%. In 2024, the scores also varied a lot—ranging from 11.39 to 15.80 points apart. This wide spread in scores highlights a major gap between students who are doing well and those who are struggling.

International benchmarks further highlight these challenges. The 2018 PISA assessment reported a mean mathematics score of 419 for Thai students, which was lower than the OECD average and representing a learning deficiency of two academic years. While 76% of students across the OECD reach Level 2 proficiency or higher, over half of Thailand's students (53%) perform below this baseline. At Level 2, students are expected to interpret and recognize mathematical representations in authentic situations without explicit instruction. By 2022, PISA scores for Thai students declined further to a mean of 394. Long-term data from 2000 to 2022 confirms a statistically significant downward trend in both mathematical and reading literacy (Institute for the Promotion of Teaching Science and Technology, 2020, n.d.; Ministry of Education, 2017; National Institute of Educational Testing Service, 2025; OECD, 2018).

To improve student literacy, contemporary now treats assessment data as the primary driver of student learning. This shift has transformed educational assessment into a rigorous discipline—one that is deeply intertwined with instructional design. We are seeing a clear move away from traditional summative judgments toward "assessment for learning." Within this framework, measurement and evaluation act as indispensable tools that allow educators to calibrate classroom activities to meet specific learning objectives. According to the recent comparisons of O-NET and PISA examination regarding content, cognitive demand, and situational context, it can be concluded that test design can actually drive instructional reform. Furthermore, the analysis of test items is vital for improving critical thinking and problem-solving. Stakeholders must prioritize identifying specific academic deficiencies and learning barriers to drive real achievement.

When examining the educational context of Chiang Mai province the educational hub of Northern Thailand it is evident that the region continues to face challenges in mathematical literacy that align with overall national trends. These challenges are particularly pronounced regarding disparities in educational quality and students' problem-solving skills. According to academic achievement data and local research in Chiang Mai, a significant number of lower secondary school students still lack the ability to connect mathematical knowledge with real-life situations. Consequently, they often develop misconceptions when tasked with solving complex contextual problems (Thongsuk, 2023). Furthermore, operational reports from local educational agencies indicate that the average mathematics scores on the Ordinary National Educational Test (O-NET) among 9th grade students in most schools across Chiang Mai remain at a level requiring urgent improvement. A major contributing factor is that students are accustomed to solving problems using fixed, rote procedures rather than developing a deep conceptual understanding. As a result, they struggle to translate real-world problem situations into mathematical models (Chiang Mai Provincial Education Office, 2023). This phenomenon presents a significant challenge, highlighting the urgent need for a systematic approach to diagnose these learning deficiencies and uncover the root causes of students' mathematical misconceptions within the region.

Accurately identifying weaknesses in mathematics requires carefully designed diagnostic test (Chanchusakun, 2018; Kaewsaiha & Chaisang, 2014). While multiple-choice tests are popular for their administrative efficiency, they also have limitations. They often fail to show a student's actual cognitive process. Crucially, these formats cannot distinguish whether a wrong answer derived from a lack of knowledge or misconceptions. To overcome this challenge, multi-tier diagnostic assessments are applied to measure more accurate cognitive depth. This started with two-tier tests, which added a "reason tier," and eventually led to four-tier models that include "confidence tiers." These extra layers help researchers verify if a student's response reflects genuine conceptual understanding or just a lucky guess.

The four-tier model is structured into four distinct levels: Tier 1 (Answer), Tier 2 (Confidence in Answer), Tier 3 (Reason), and Tier 4 (Confidence in Reason). The strength of this model lies in its ability to categorize student understanding. For example, an incorrect response paired with high confidence reveals a specific misconception. Conversely, a correct response accompanied by high confidence signifies true conceptual mastery (IEEE, 2025; Li et al., 2024; Tophochanapan, 2022).

To improve mathematical literacy, we must prioritize the development of student problem-solving capabilities. Most mathematical literacy assessments focus heavily on this specific process. It requires the ability to formulate real-world issues in mathematical terms, apply relevant concepts to find a solution, and then interpret those results to solve everyday problems. Any error or deficiency in this process will directly lower a student's assessment outcomes. To address this, our research investigates the current state of mathematical literacy gaps among 9th grade students in Chiang Mai Province. The researchers have developed a four-tier diagnostic test specifically for this purpose. The data from this instrument will inform instructional reforms, ultimately empowering students to reach higher levels of problem-solving proficiency.

Objective

1. To investigate the current state of deficiencies in mathematical literacy for 9th grade students in Chiang Mai Province
2. To construct a specific diagnostic test for 9th grade students in Chiang Mai Province

Conceptual Framework

The conceptual framework of this study, titled "Construction of a Mathematical Literacy Diagnostic Test for 9th Grade Students in Chiang Mai Province," the researcher developed a conceptual framework based on a comprehensive scope of mathematics literacy diagnostic tests and four-tier diagnostic tests. The framework is highlighted as follows:

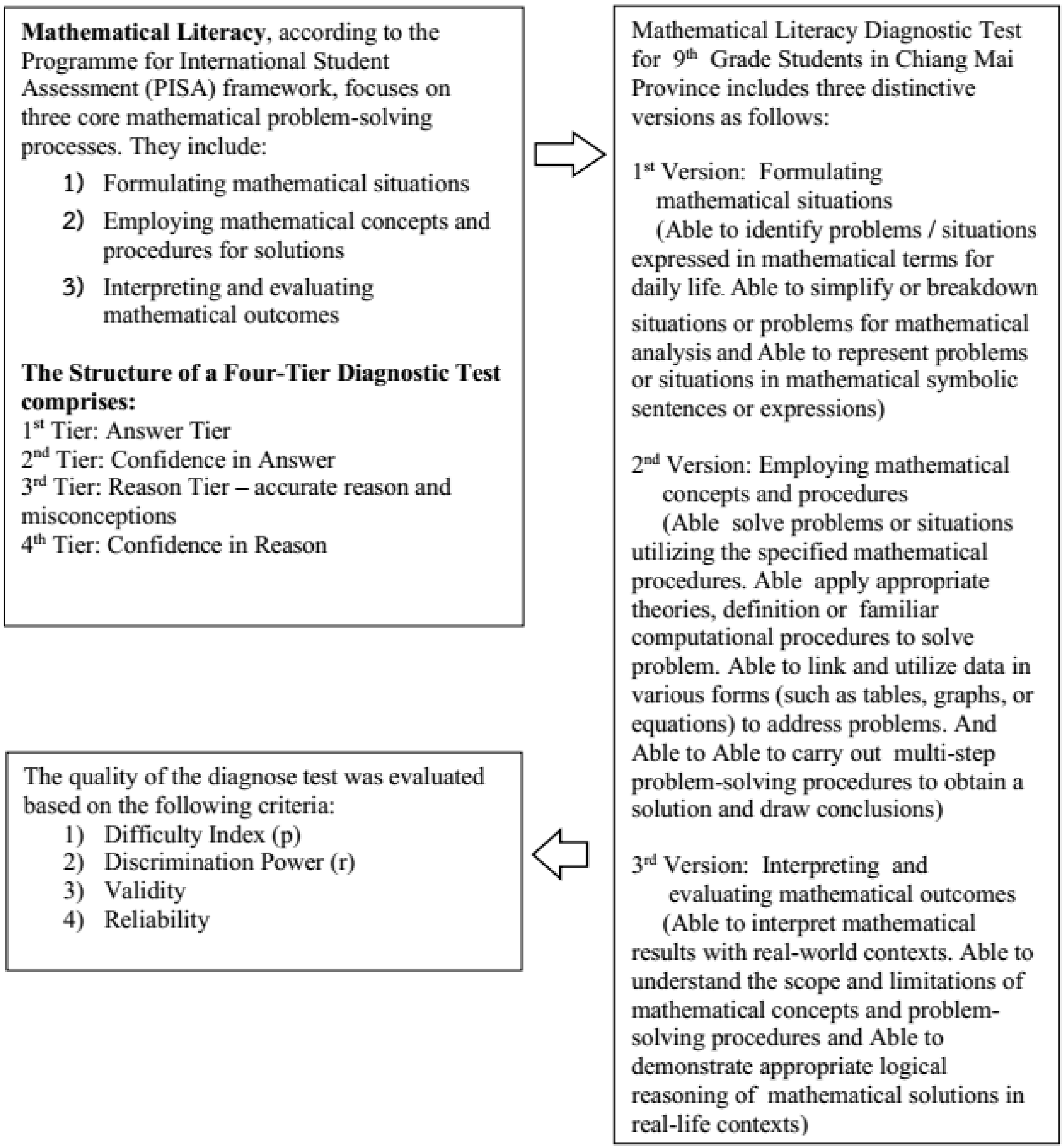

Figure 1 Conceptual Framework

Research Methodology

The research methodology is divided into two phases as the following details.

Research Sample

Phase 1 Sample Group: 385 of 9th grade students in Chiang Mai Province, selected by a multi-stage random sampling technique to mathematical literacy deficiency survey test.

Phase 2 Sample Group: 100 of 9th grade students in Chiang Mai Province, also obtained by a multi-stage random sampling to four-tier mathematical literacy diagnostic test.

Research Instruments

Phase 1 involved a mathematical literacy deficiency survey test. The test is an open-ended format comprising two tiers: the answer tier and the reason tier. and a content validity verification instrument.

Phase 2 involved a four-tier mathematical literacy diagnostic test. The instrument used in this study is a four-tier diagnostic test consisting of four tiers: the answer tier, the confidence of answer tier, the reason tier, and the confidence of reason tier., along with a content validity verification instrument and a criterion-related validity verification instrument.

Data Collection

Phase 1: Investigating the State of Mathematical Literacy Deficiencies among 9th Grade Students in Chiang Mai Province. The procedures were as follows:

1) The researchers reviewed existing mathematical literacy frameworks to conceptualize the problem-solving process within the specified context. This process refers to the sequence of actions through which individuals connect real-world situations with mathematical principles. It involves three key stages: formulating mathematical situations, applying relevant concepts and procedures, and interpreting results. Based on this review, ten behavioral indicators were identified to represent students' problem-solving processes.

2) the derived indicators were used to develop a test blueprint and to construct a survey instrument for investigating learning deficiencies. The test content covered four mathematical domains: quantity, space and shape, change and relationships,

and uncertainty—examined across four contextual categories: personal, occupational, social, and scientific.

3) A panel of five experts reviewed the draft for content validity. This process involved calculating the Index of Item-Objective Congruence (IOC) to ensure each item aligned with the intended learning outcomes.

4) The test items were revised based on the experts' recommendations. The items with an IOC value greater than 0.50 were retained for the final survey test.

5) The survey test was administered to the first sample group, consisting of 385 students, to investigate their mathematical literacy deficiencies. The test was divided into two sets, each consisting of 30 items, for a total of 60 items to prevent testing fatigue and maintain the results' validity. Students were allocated 60 minutes per set for completion.

6) Students' incorrect responses were collected and analyzed to develop authentic distractors for a subsequent diagnostic test. When more than three incorrect response patterns were identified, the three most frequent errors were selected as distractors. When fewer than three errors occurred, additional distractors were constructed based on commonly observed misconceptions. These errors were further classified into specific deficiency types and used to develop the reasoning tier for each item, enabling a more precise diagnosis of students' learning difficulties.

Phase 2: Constructing the Four-Tier Mathematical Literacy Diagnostic Test for 9th Grade Students in Chiang Mai Province. The procedures were as follows:

1) The four-tier multiple-choice diagnostic test was constructed, consisting of the Answer Tier, the Confidence Tier for the answer, the Reason Tier, and the Confidence in reason Tier. The frequent incorrect responses obtained from the Phase 1 survey were create as distractors for both the Answer Tier and the Reason Tier in each test item.

2) A panel of five experts evaluated the draft for content validity. All three versions were determined the Index of Item–Objective Congruence (IOC) between the test items, response options, and underlying reasons and the specified behavioral indicators.

3) According to the experts' recommendations, the researchers revised the test items, response options, and reasons. Items with an IOC value higher than 0.50

were selected for complying into the final four-tier multiple-choice diagnostic test for mathematical literacy deficiencies.

4) The four-tier multiple-choice diagnostic test was tired out with 100 students as the second sample group. The researchers explained all details of the test, response methodology, and its format. Each item consisted of four integrated components: the Answer Tier (A-tier), the Confidence Tier for the answer (CA-tier), the Reason Tier (R-tier), and the Confidence Tier for the reason (CR-tier). Subsequently, the collected data were analyzed the quality of both individual items and the overall test according to criterion-referenced item analysis.

Summary of the Study

The State of Mathematical Literacy Deficiencies among 9th Grade Students in Chiang Mai Province.

The construction of the Mathematical Literacy Diagnostic Test for 9th students in Chiang Mai Province involved the review of the mathematical literacy assessment framework, analyzed, and defined the scope of mathematical processes, and constructed a test blueprint, which was utilized to investigate learning deficiencies. A panel of experts then reviewed for content validity with IOC values ranging from 0.80 to 1.00. After that, the test items were revised in accordance with the experts' recommendations. It had six situational contexts and 60 items and was administered to a sample of 385 students. The incorrect responses were collected from to identify mathematical literacy deficiencies across each behavioral indicator, as presented in Table 1.

Table 1: Mathematical literacy deficiencies categorized by mathematical problem-solving processes and behavioral indicators

Mathematical Problem-Solving Processes	Behavioral Indicators	Mathematical Literacy Deficiencies
Formulating mathematical situations	Able to identify problems / situations expressed in mathematical terms for daily life	Cannot explain the essential components of problem
		Cannot identify mathematical problems embedded within given tasks
		Cannot indicate the key variables in solving problems
	Able to simplify or breakdown situations or problems for mathematical analysis	Unable to determine the mathematical methods for solution
		Unable to identify the relationships between the problem and the provided data
	Able to represent problems or situations in mathematical symbolic sentences or expressions	Unable to translate word problems into mathematical symbolic sentences
		Unable to apply computational methods to visualize mathematical relationships
Employing mathematical concepts and procedures for solutions	Able solve problems or situations utilizing the specified mathematical procedures	Unable to substitute values for variables in a mathematical equation
		Unable to perform simple calculations
	Able apply appropriate theories, definition or familiar computational procedures to solve problem	Unable to select appropriate procedures or data in solving problems
		Unable to perform successful problem-solving procedures
	Able to link and utilize data in various forms (such as tables, graphs, or equations) to address problems	Unable to link available information to solve problems
		Unable to connect the prior knowledge to new knowledge in solving problems
		Unable to solve problems in sequential order
	Able to Able to carry out multi-step problem-solving procedures to obtain a solution and draw conclusions	Unable to apply multi procedures to solve problems
		Unable to interpret mathematical results in real-life contexts
Interpreting and evaluating mathematical outcomes	Able to interpret mathematical results with real-world contexts	Unable to provide supporting reasoning when interpreting results in everyday life contexts
		Unable to indicate the limitations of in formation in everyday life
	Able to understand the scope and limitations of mathematical concepts and problem-solving procedures	Unable to explain what limitations are in the real world
		Unable to provide reasoning to support or challenge mathematical information presented in a problem
	Able to demonstrate appropriate logical reasoning of mathematical solutions in real-life contexts	Unable to identify the methods fail to reflect real-world reality
		Unable to take into account the differences between two distinct problems

As shown in Table 1 demonstrates the alignment among the three core mathematical problem-solving processes, the 10 behavioral indicators, and the 22 identified mathematical literacy deficiencies

Results of Constructing the Four-Tier Mathematical Literacy Diagnostic Test for 9th Grade Students in Chiang Mai Province and Evaluation of the Test Quality

The results of constructing the Mathematical Literacy Diagnostic Test for 9th grade students in Chiang Mai applied a four-tier diagnostic format consists of: (1) Answer Tier, (2) Confidence in Answer, (3) Reason Tier (comprising correct reasons and misconceptions), and (4) Confidence in Reason. The test was constructed based on established mathematical problem-solving processes, which consist of three stages: formulating mathematical situations, applying mathematical concepts and procedures, and interpreting and evaluating outcomes. The content domains were aligned with the mathematics literacy assessment framework of the Programme for International Student Assessment (PISA). Detailed information regarding the test structure and content alignment is presented in Table 2.

Table 2: Showing the number of items for each test version

Version	Number of Indicators	Number of Test Items
1th Version: Formulating mathematical situations	3	15
2nd Version: Employing mathematical concepts and procedures for solutions	4	20
3rd Version: Interpreting and evaluating mathematical outcomes	3	15

As shown in Table 2, the instrument consists of three tests comprising 3, 4, and 3 indicators, with 15, 20, and 15 items, respectively.

2) Results of content validity (IOC), item-objective congruence, criterion-referenced item and test quality, and concurrent validity of a mathematical literacy diagnostic test for 9th grade students in Chiang Mai Province were presented as in Table 3.

Table 3: Item-level and test quality analysis for criterion-referenced assessments, and concurrent validity as evaluated by expert teachers familiar with the sample group

Version	IOC	Difficulty Index (p)	Discrimination Power (r)	Reliability	Validity
1st Version	.60 – 1.0	.21 - .71	.20 - .58	.86	.82
2nd Version	.60 – 1.0	.24 - .79	.20 - .72	.89	.82
3rd Version	.60 – 1.0	.22 - .76	.20 - .48	.78	.83

As shown in Table 3, the criterion-referenced quality of the individual items and the overall assessment across the three tests revealed that the Item-Objective Congruence (IOC) index ranged from .60 to 1.0, the difficulty index from .21 to .79, the discrimination power from .20 to .72, the reliability from .78 to .89, and the validity from .82 to .83.

Summary of the Study

The study on mathematical literacy deficiencies was conducted by developing a diagnostic test derived from the three mathematical processes. The results revealed 22 types of deficiencies across 10 behavioral indicators. The development of the mathematical literacy diagnostic test for 9th grade students in Chiang Mai province was in a four-tier diagnostic format, resulting in three test versions: (1) formulating mathematical situations (15 items), (2) employing mathematical concepts and procedures (20 items), and (3) interpreting and evaluating mathematical outcomes (15 items). The results pointed out that the item difficulty indices ranged from 0.21 to 0.79, and the discrimination indices ranged from 0.20 to 0.72. The reliability coefficients of the three test versions were .86, .89, and .78, respectively, while the concurrent validity scores were 0.82, 0.82, and 0.83. In addition, the criterion-related validity coefficients of the three test forms were .82, .82, and .83, respectively.

Discussions

1. State of mathematical literacy deficiencies: The findings from Phase 1 revealed that 9th-grade students exhibited deficiencies in mathematical literacy across three core processes: Formulating situations mathematically, Employing mathematical concepts, facts, procedures, and reasoning, and Interpreting, applying, and evaluating mathematical outcomes. A total of 22 specific deficiency characteristics were identified from 10 behavioral indicators.

These deficiencies reflect limitations in mathematical problem-solving, which align with the reports of Thailand's PISA assessments in 2018 and 2022. The reports indicated that the average mathematics scores of Thai students have shown a declining trend and are significantly lower than the OECD average. Notably, over 53% of the students performed below Level 2 (the baseline proficiency level). This phenomenon underscores that students continue to face obstacles in connecting real-world contexts with mathematical knowledge. They are unable to translate problem situations into mathematical models or symbolic sentences, and they lack the skills to interpret outcomes and apply them back to explain the original problem contexts.

A crucial reason why these research findings closely align with the issues highlighted in the PISA assessments is the nature of the evaluation process, which emphasizes "Higher-Order Thinking Skills" rather than mere "Procedural Knowledge." Considering their prior learning contexts, students are often accustomed to solving routine problems with explicit numbers and instructions but lack practice in "Mathematical Modelling" skills. Consequently, they experience cognitive stagnation when confronted with ambiguous and complex real-world problem situations. The findings of this study, therefore, confirm that students are unable to identify mathematical problems from given contexts and cannot interpret meanings for practical application in daily life. These aspects are recognized as the core of Mathematical Literacy, which focuses on applying knowledge to solve problems in diverse situations (OECD, 2018).

Furthermore, these findings are consistent with the research conducted by Kamonthip Toopkham (2022), which examined deficiencies in solving mathematical word problems. The study found that students were unable to analyze the meaning of word problems to distinguish between the given information and what the problem required them to find. Additionally, they could not convert the information and questions into variables, ultimately leading to errors in the substitution and calculation processes. This connection can be explained by cognitive constraints, highlighting that the "translation from natural language to mathematical language" acts as a critical bottleneck hindering the students' learning process. If students cannot overcome the initial hurdle of interpreting and defining variables, it inevitably triggers a chain reaction, resulting in deficiencies in the subsequent processes of both Employing and Interpreting.

2. Mathematical Literacy Diagnostic Test: The results of Phase 2 in constructing three versions of the four-tier diagnostic test the good quality. The difficulty indices (p) ranged from 0.21 to 0.79, indicating that the test items had appropriate levels of difficulty. The discrimination indices (r) ranged from 0.20 to 0.72. In addition, the reliability coefficients of the three test forms were .86, .89, and .78, respectively, indicating high reliability. The concurrent validity coefficients were also high, with values of .82, .82, and .83 for the three test versions. The findings indicate that the four-tier diagnostic format is a highly effective tool for assessing and diagnosing learning deficiencies. This is consistent with the frameworks of Li et al. (2024), who mention that the inclusion of confidence tiers provides essential verification. Consequently, it distinguishes whether a student's incorrect response stems from a lack of knowledge or misconceptions.

The high reliability and validity of this test suite signify that the instrument can distinguish student cognitive processes with substantially greater precision than conventional multiple-choice assessments. These results align with research by Topochanapan (2022), who used a four-tier diagnostic test to show that adding "confidence tiers" helps stop students from simply guessing. This approach allows students to be more effectively classified according to their patterns of thinking. Crucially, this assessment can tell the difference between a student with a specific misconception and one who just doesn't know the answer. Standard multiple-choice tests usually miss this distinction entirely. Because the tool shows such high reliability, it serves as a dependable way to measure what a student truly understands, moving beyond the surface-level results of traditional testing. Furthermore, the high reliability of this instrument suggests that a four-tier structure can consistently capture a student's true competency and authentic understanding, rather than merely recording surface-level responses. Furthermore, the findings are consistent with the research of Chiranorawanich (2021), who constructed a four-tier diagnostic test on exponents and reported content validity (IOC) ranging from 0.60 to 1.00, with item difficulty and discrimination power within acceptable standards. This confirms that the test development process is initiating from a 'Survey Test' to identify student deficiencies and utilizing them as 'Distractors' in the actual diagnostic test. It is a highly effective methodology. This test can accurately diagnose students' learning difficulties in authentic learning contexts. Chiranorawanich (2021) also worked on this by creating a four-tier diagnostic test specifically for Algebra. By sorting student understanding into four clear groups—scientific knowledge, Lack of Confidence, Misconceptions, and Lack of Knowledge—this tool does more than just grade a test. It pinpoints the specific gaps in a student's logic, especially where they hold deep-seated misconceptions. This focused support led to a clear boost in academic performance for the students in the 'Misconception' group.

Recommendations

Based on the summary and discussion of the research findings, the researcher provides the following recommendations for the practical application of the results and suggestions for future research.

1. Recommendations for Implementation

   Application for Remedial Instruction: educational institutions and 9th grade mathematics teachers should apply these three versions of the Mathematical Literacy Diagnostic Test as tools for pre- test and post-test. This test is effective to categorize students who are just guessing from those who have real misunderstandings. Identifying these specific struggles allows teachers to move past general lessons and build strategies that target actual learning gaps. This method promotes more focused teaching and facilitates students make progress more efficiently.

2. Recommendations for Future Research

   Since this study focuses on diagnosis of learning deficiencies, future research should aim to develop and implement instructional innovations to address the misconceptions on 22 areas identified in this study. In particular, it should extend the findings toward the creation of effective innovations for problem remediation.

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